DRAFT SYLLABUS UNDER AUTONOMY

MATHEMATICS

SEMESTER I **COURSE : S.MAT.1.01**

CALCULUS AND ANALYTIC GEOMETRY I **[60 LECTURES]**

LEARNING OBJECTIVES :

To learn about i) Intermediate value theorem
ii) Curve Tracing
iii) Mean Value Theorems

Prerequisites

(i) Limits of some standard functions as x approaches a (a $\in$ R), such as constant fuction, x , x^n, sin x , cos x , tan x , $\frac{x^n - a^n}{x-a}$ $n \in Q$ $(a > 0)$ exponential and logarithmic functions, $\lim_{x \to 0} \frac{\sin x}{x}$ continuity in terms of limits.

(ii) Derivatives, Derivatives of standard functions such as constant function, x^n, trigonometric functions, e^x , a^x $(a > 0)$, $\log x$.

UNIT 1. Limit and continuity of functions of one variable [20 lectures]

(a) (i) Absolute value of a real number and the properties such as |- a| = |a|, |ab| = |a| |b| and |a+b| $\leq$ |a| + |b|.

(ii) Intervals in R, neighbourhoods and deleted neighbourhoods of a real number, bounded subsets of R.

(b) (i) Graphs of functions such as $|x|, \frac{1}{x}, ax^2 + bx - c, [x]$ (flooring function), [x] (Ceiling function), x^3, sin x , cos x , tan x , sin $\frac{1}{x}$, $x \sin \frac{1}{x}$ over suitable intervals.

(ii) Graph of a bijective function and its inverse. Examples such as x^2 and $x^{1/2}$, x^3 and $x^{1/3}$, a x + b (a $\neq$ 0) and $\frac{1}{a}x - \frac{b}{c}$ over suitable domains curve tracing.

(c) (i) Statement of rules for finding limits, sum rule, difference rule, product rule, constant multiple rule, quotient rule.
(ii) Sandwich theorem of limits (without proof).
(iii) Limit of composite functions (without proof).

(d) $\in -\delta$ definition of limit of a real valued function, simple illustrations lime $ax+b, \sqrt{x+a}$ (a $\geq$ 0), x^2, sin x, cos x. (In general, evaluation of limits to be done using rules in (c)). $\in -\delta$ definition of one sided limit of a real valued function. Formal definition of infinite limits, examples such as $\lim_{x \to 0^+} \frac{1}{x}, \lim_{x \to 0^-} \frac{1}{x}$.

(e) (i) Continuity of a real valued function at a point in terms of limits, and two sided limits. Graphical representation of continuity of a real valued function.
(ii) Continuity of a real valued function at end points of domain.
(iii) Removable discontinuity at a point of a real valued function and extension of a function having removable discontinuity at a point to a function continuous at that point.
(iv) Continuity of polynomials and rational functions.
(v) Constructing a real valued function having finitely many prescribed points of discontinuity over an interval.

(f) Continuity of a real valued function over an interval. Statements of properties of continuous functions such as the following:
(i) Intermediate value property.
(ii) A continuous function on a closed and bounded interval is bounded and attains its bounds.

Elementary consequences such as if $f:[a,b] \to R$ is continuous then range of f is a closed and bounded interval.
Applications.

(g) Definition of limit as x approaches $+\infty$, examples. Limits of rational function as x approaches $+\infty$.

Reference for Unit 1: Chapter Preliminaries, Section 1 and Chapter 1, Sections 1.2, 1.3, 1.4, 1.5 of Calculus and Analytical Geometry, G. B. THOMAS and P. L. FINNEY, Ninth Edition, Addison – Wesley, 1998.

UNIT 2. Differentiability of functions of one variable [20 Lectures]

(a) Definition of derivative of a real valued function at a point, notion of differentiability, geometric interpretation of a derivative of a real valued function at a point, differentiability of a function over an interval, statement of rules of differentiability, chain rule of finding derivative of composite differentiable functions, derivative of an inverse function (without proof). Intermediate value property of derivative (without proof) and its applications, implicit differentiation.

(b) (i) Differentiable functions are continuous, but the converse is not true.

(ii) Higher order derivatives, examples of functions $x^n |x|$, n = 0, 1, 2,… which are differentiable n times but not (n + 1) times.

(iii) Leibnitz Theorem for n^{th} order derivative of product of two n times differentiable functions.

Reference for Unit 2: Chapter 2. Sections 2.1, 2.2, 2.3, 2.5, 2.6 of Calculus and Analytic Geometry, G. B. THOMAS and R. L. FINNEY, Ninth Edition, Addison-Wesley, 1998. And Chapter 4, Section 4.1 of A Course in Calculus and Real Analysis, SUDHIR R. GHORPADE and BALMOHAN V. LIMAYE, Springer International Edition.

UNIT 3. Applications of derivatives [20 Lectures]

(a) (i) Mean Value Theorems : Rolle's Mean Value Theorem, Lagrange's Mean Value Theorem, Cauchy's Mean Value Theorem.

(ii) Taylor's polynomial and Taylor's Theorem with Lagrange's form of remainder.

(b) Extreme values of functions, absolute and local extreme critical points, increasing and decreasing functions, the second derivative test for extreme values.

(c) Graphing of functions using first and second derivatives, the second derivative test for concavity, points of inflection.

(d) Limits as x approaches $+\infty$. Asymptotes – horizontal and vertical.

Reference for Unit 3: Chapter 3, Sections 3.1, 3.2, 3.4, 3.5 and Chapter 8, Section 8.9 of Calculus and Analytic Geometry, G. B. THOMAS and R. L. FINNEY, Ninth Edition, Addison-Wesley, 1998, and Chapter 4, Sections 4.2, 4.3, 4.4 and Chapter 5, Sections, 5.1, 5.2 of A Course in Calculus and Real Analysis, SUDHIR R. GHORPADE and BALMOHAN V. LIMAYE, Springer International Edition.

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REFERENCES:

1. G. B. THOMAS and R. L. FINNEY, Calculus and Analytic Geometry, Ninth Edition, Addition-Wesley, 1998.

2. SUDHIR R. GHORPADE and BALMOHAN V. LIMAYE, A Course in Calculus and Real Analysis, Springer International Edition.

3. HOWARD ANTON, Calculus – A new Horizon, Sixth Edition, John Wiley and Sons Inc., 1999.

4. JAMES STEWART, Calculus, Third Edition, Brooks/Cole Publishing Company, 1994.

Additional Reference Books

1. TOM APOSTOL, Calculus Volume 1, One variable calculus with an introduction to Linear Algebra, Second Edition, Wiley Publications. (Evaluation copy available for instructors).

2. DEBORAH HUGHES-HALLET, ANDREW GLEASON, DANIEL FLATH, PATTI FRAZER, THOMAS TUCKER, DAVID LOMEN, DAVID LOVELOCK, DAVID MUMFORD, BRAD OS-GOOD, DOUGLAS QUINNEY, KAREN RHEA, JEFF TECOSKY-FREDMAN, Calculus, Single and Multivariable, Fourth Edition, Wiley Europe Higher Education (Online version of textbook and online grade book available).

3. SATUNIO I. SALAS, GARRET J. EIGEN, EINAR HILLE, Calculus: One variable, Wiley Europe Higher Education (online version and online grade book available).

Suggested topics for Tutorials/Assignments

(1) Graphs and functions.

(2) Limits of functioning using $\in -\delta$ definition. (Simple functions recommended in the syllabus)

(3) Calculating limits using rules of limit and Sandwich theorem.

(4) Continuity over an interval (intermediate value property, maxima, minima over closed interval).

(5) Differentiability, chain rule, implicit differentiation.

(6) Higher order derivatives and Leibnitz theorem.

(7) Mean Value Theorems Applications.

(8) Taylor's Theorem. Taylor's polynomials.

(9) Extremas; Local and absolute.

(10) Graphing of functions, Asymptotes.

(11) Vectors in R^2 and R^3, Cartesian, spherical and cylindrical co-ordinates in R^3.

(12) Polar graphing.

(13) Level curves of functions of two variables.

(14) Limits and continuity at a point of functions of two variables, calculating limits, where they exist, and non-existence of limits using two path test.

(15) Partial derivatives of first order, second order.

(16) Differentiability of a function at a point, linearization.

(17) Derivatives of composite functions.

(18) Directional Derivatives, gradient, tangent planes.

(19) Extreme values using second derivative test and Langrage's multiplier method.

SEMESTER I COURSE : S.MAT.1.02

DISCRETE MATHEMATICS I [60 LECTURES]

LEARNING OBJECTIVES :

To learn about i) Euclidian algorithm
ii) Properties of Congruences
iii) Pigeon hole principle

Prerequisites

(i) Set Theory: Set, subset, union and intersection of two sets, empty set, universal set, complement of a set, De Morgan's laws, Cartesian product of two sets.
(ii) Relations, functions
(iii) First Principal of finite induction.
(iv) Permutations and combinations, ${}^nC_{r,}$ ${}^nP_r$
(v) Complex numbers: Addition and multiplication of complex numbers, modulus, amplitude and conjugate of a complex number.

Unit 1. Integers and divisibility [20 lectures]

a) (i) Statements of well-ordering property of non-negative integers.
(ii) Principle of finite induction (first and second) as a consequence of well-ordering property

Binomial Theorem for non-negative exponents, Pascal Triangle. Recursive definitions.

b) Divisibility in integers, division algorithm, greatest common divisor (g.c.d.) and least common multiple (l.c.d.) of two integers, basic properties of g.c.d. such as existence and uniqueness of g.c.d. of integers a and b and that the g.c.d. can be expressed as ma + nb for some $m, n \in \mathrm{Z}$, Euclidean algorithm.

c) Primes, Euclid's lemma, Fundamental Theorem of Arithmetic.

Reference for Unit 1: Chapter 1, Sections 1.1, 1.2, Chapter 2, Sections 2.1, 2.2, 2.3, and Chapter 3, Section 3.1 of Elementary Number Theory, DAVID M. BURTON, Second Edition, BS, New Delhi.

Unit 2. Functions and Counting Principles **[20 Lectures]**

(a) (i) Review of functions, domain and range of a function, composite functions.
(ii) Direct image f (A) and inverse image $f^{-1}(B)$ of a function f.
(iii) Injective, surjective, bijective functions. Composite of injective, surjective, bijective functions are injective, surjective and bijective respectively.
(iv) Invertible functions and finding their Inverse. Bijective functions are invertible and conversely.
(v) Examples of functions including constant, identity, projection, inclusion.

(b) Binary operation as a function, properties, examples. Definition and examples of groups and subgroups.

(c) (i) There is no injection from N_n to N_m if n > m, $N_n = \{1,2, \ldots, n\}$
(ii) Pigeon Hole Principle and its applications.
(iii) Finite and infinite sets, cardinality of a finite set.
(iv) The number of subsets of a finite set having n elements is 2^n

Reference for Unit 2: Chapter 2 of Discrete Mathematics, NORMAN L. BIGGS, Revised Edition, Clarendon Press, Oxford 1989.

Unit 3. Integers and Congruences **[20 lectures]**

(a) (i) The set of primes is infinite
(ii) The set of primes of the type 4n – 1 (or 4n + 1 or 6n – 1) is infinite.

(b) Congruences: Definition and elementary properties.

(c) Euler φ function, invertible elements modulo n

(d) (i) Euler's Theorem (without proof).
(ii) Fernat's Little Theorem.
(iii) Wilson's Theorem
(iv) Applications: Solution of linear congruences.

Reference for Unit 3: Chapter 4, Chapter 5, Sections 5.2, 5.3, 5.4 and Chapter 7, Sections 7.1, 7.2, 7.3 of Elementary Number Theory. DAVID M. BURTON. Second Edition. UBS, New Delhi.

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SEMESTER II COURSE : S.MAT.2.01

CALCULUS AND ANALYTIC GEOMETRY II [60 LECTURES]

LEARNING OBJECTIVES :

To learn about i) Quadric surfaces
ii) Partial derivatives and directional derivatives
iii) Extremization of two variables

Unit 1. Analytic Geometry in Euclidean spaces [20 Lecture]

(a) Review of vectors in R^2, R^3, component form of vectors, basic notions such as addition and scalar multiplication of vectors, dot product of vectors, orthogonal vectors, length (norm) of a vector, unit vector, distance between two vectors, cross product of vectors in R^3, scalar triple product (box product), vector projections.

(b) Lines and planes in space, equation of sphere, cylinders and quadric surfaces:

(c) Polar co-ordinates in R^2, polar graphing with examples like $r = \sin \theta$, $r = \cos 2\theta$, $r = a(1 - \cos \theta)$.

(d) Relationship between polar and Cartesian co-ordinates in R^2, cylindrical and spherical co-ordinates in R^3 and relationships of these co-ordinates with cartesian co-ordinates and each other.

Reference for Unit 1: Chapter 9, Sections 9.4, 9.6, 9.7, Chapter 10 of Calculus and Analytical Geometry. G. B. THOMAS and R. L. FINNEY, Ninth Edition, Addison-Wesley, 1998.

Unit 2. Limits and continuity of functions of two and three variables. [20 Lectures]

(a) (i) Open disc in R^2 and R^3, boundary of open disc, closed disc in R^2 and R^3, bounded regions, unbounded regions in R^2 and R^3.
(ii) Real valued functions of two or three variables, examples. Level curves for functions of two variables. Use of level curves to draw graphs of

z = f (x, y), especially quadric surfaces. $\in - \delta$ definition of a limit of a real valued function of two variables (only brief statement).

(b) Statement of rules of limits in two (or three) variables. Sum rule, difference rule, product rules, constant multiple rule, quotient rule, power rule.
Applying these rules to determine limits of polynomial and rational functions. Definition of continuity of functions of two (or three) variables in terms of limits.

(c) Definition of a path. Limit of a function along paths. Two path test for non-existence of a limit. Examples of functions such as $\frac{2xy}{x^2+y^2}, \frac{2x^2y}{x^4+y^2}, \frac{-xy}{x^2+y^2}$ etc. Sandwich theorem for a function of two variables (without proof).

(d) Calculator $\lim_{(x,y)\to(0,0)} f(x,y)$ by changing to polar co-ordinates (illustrating with examples).

(e) Vector valued functions of one and several variables, planar and space curves, component functions, vector fields, graphs of vector valued function like (cos t, sin t), (cos t, sin t, 1). (cos t, sint, t). Limits of vector valued function by taking limits of component functions.

Reference for Unit 2: Chapter 12, Sections 12.1, 12.2 of Calculus and Analytic Geometry, G. B. THOMAS and R. L. FINNEY, Ninth Edition, Addison-Wesley, 1998.

Unit 3. Differentiability of functions of two variables [20 Lectures]

(a) (i) Partial derivatives of a real valued function of two variables, the relationship between continuity and the existence of partial derivatives at a point. Second order partial derivatives, Mixed derivative theorem for two variables (without proof). The increment theorem for two variables (without proof).
(ii) Differentiability of a function of two variables at a point over a disc, linearization of a differentiable function at a point.
(iii) Chain rule for composite function of the type $R^2 \xrightarrow{f} R \xrightarrow{g} R$ (without proof).
(iv) Implicit differentiation.

(b) Directional derivatives in a plane, interpretation of directional derivatives, gradient vector, relation between directional derivative and gradient.

(c) Geometric interpretation of partial derivatives and its relation to the tangent plane at a point.

(d) Extreme values of a function of two variables. Local maximum, local minimum and first derivative test for local extreme values (without proof). Critical points, saddle points, second derivative test for local extreme values (without proof).

(e) The method of Lagrange's Multiplier to obtain extrema of a function of two variables (one constraint only).

(f) Derivative of vector valued function as derivative of component functions. Statement of rules of differentiation – sum, difference, product, constant multiple. Chain rule for composite function of the type $R \xrightarrow{f} R^2 \xrightarrow{g} R$ (without proof). Geometric interpretation of derivatives. Derivative of dot and cross products.

Reference for Unit 3: Chapter 12, Sections 12.3, 12.6, 12.7, 12.8, 12.9 of Calculus and Analytical Geometry, G. B. THOMAS and R. L. FINNEY, Ninth Edition, Addison-Wesley, 1998.

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NOTE:

1. It is recommended that the concepts may be illustrated using computer software and graphing calculators wherever possible.

2. Applications and mathematical modeling of concepts to be done wherever possible.

REFERENCES:

1. G. B. THOMAS and R. L. FINNEY, Calculus and Analytic Geometry, Ninth Edition, Addition-Wesley, 1998.
2. SUDHIR R. GHORPADE and BALMOHAN V. LIMAYE, A Course in Calculus and Real Analysis, Springer International Edition.
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(7) Mean Value Theorems Applications.

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(16) Differentiability of a function at a point, linearization.

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SEMESTER II **COURSE : S.MAT.2.02**

DISCRETE MATHEMATICS II **[60 LECTURES]**

LEARNING OBJECTIVES :

To learn about i) Transpositions
ii) Partition of a set
iii) Fundamental theorem of Algebra

Unit 1. Principles of Counting and Equivalence Relations **[20 lectures]**

(a) (i) Addition and multiplication Principles.
(ii) Counting sets of pairs.

(b) Cartesian product of n sets.

(c) Partition of a set, S (n, k), the Sterling numbers of second kind defined in terms of partitions. Properties of S (n, k).

(d) (i) Equivalence relation. Equivalence classes. Properties. Examples including the relation modulo n on Z.

(ii) An equivalence relation induces partition of a set and a partition of a set defines an equivalence relation.

(e) (i) Equivalent sets, countable, uncountable sets.
(ii) Examples of countable sets including Z, Q^+ (the set of positive rational numbers), Q, N x N.
(iii) Examples of uncountable sets including (0, 1), R, (a,b)
(iv) A set is not equivalent to its power set.

Reference for Unit 1: Chapter 3. Section 3.1, 3.2, 3.3 and Chapter 5, Sections 5.1, 5.2 of Discrete Mathematics, NORMAN L. BIGGS, Revised Edition, Clarendon Press, Oxford 1989.

Unit 2. Principles of Counting and permutations **[20 lectures]**

(a) Distribution of objects, multinomial numbers, combinatorial interpretations, multinomial theorem.
(b) Inclusion and Exclusion (Sieve, Principle)
(c) (i) Number of functions from a finite set X to a finite set Y
(ii) Number of injective functions from a finite set X to a finite set Y, where $|X| \leq |Y$
(iii) Number of subjective functions from a set X to a finite set Y, where $|X| \geq |Y$

(d) Permutations:
(i) Permutations on n symbols. The set S_n and the number of permutations in S_n is nil

(ii) Composition of two permutations as a binary operation in Sn composition on permutation is non-commutative if $n \geq 3$.

(iii) Cycles and transpositions, representation of a permutation as product of disjoint cycles. Listing permutations in S3, S4, etc.

(iv) Sign of a permutation, sign of transposition is -1, multiplicative property of sign, odd and even permutations, number of even permutations, number o f even permutations in Sn is $\frac{-1}{2}$

(v) Partition of a positive integer, its relation to decomposition of a permutation as product disjoint cycles, conjugate of a permutation.

(e) (i) Derangements on n symbols, d_n, the number of derangements of {1,2, …n}
(ii) Arithmetic applications including Euler-φ function.

(f) Recurrence relations: formulation and solutions. Solving, homogeneous linear recurrence relations.

Reference for Unit 2: Chapter 3, Sections 3.3, 3.5, 3.6 and chapter 5, Sections 5.3, 5.5, 5.6 of Discrete Mathematics, NORMAN L. BIGGS, Revised Edition, Clarendon Press, Oxford 1989 and Chapter 3, Sections 3.3, 3.5, 3.6 and Chapter 5, Sections 5.3, 5.5, 5.6 of Discrete Mathematics and its applications, KENNETH H. ROSEN, McGraw Hill International Edition, Mathematics Series.

Unit 3. Polynomials **[20 lectures]**

(a) (i) The set F [X] of polynomials in one variable over F where F = Q. R, or C. Addition, Multiplication of two polynomials, degree of a polynomial, basic properties.

(ii) Division algorithm in F[X] (without proof) and g.c.d. of two polynomials, and its basic properties (without proof), Euclidean algorithm, (without proof), applications.

(iii) Roots of a polynomial, multiplicity of a root, Factor theorem. A polynomial of degree n over F has at most n roots. Necessary condition for $p \geq \epsilon$ Q to be a root of polynomial in Z[X]

(iv) Statement of Fundamental Theorem of Algebra and its elementary consequences.

(v) Complex number z is a root of a polynomial f(X) over R if and if its conjugatge $\bar{z}$ is a root of f (X).

(vi) Polynomials in R[X] can be expressed as a product of linear and quadratic polynomial in $\geq$[X].

(vii) Statement of De Morgan's Theorem, n^{th} roots of unity, Primitive n^{th} roots of unity, n^{th} roots of a complex number ά

Reference for Unit 3: Chapter 15, Sections 15.4, 15.5, 15.6 of Discrete Mathematics, NORMAN L. BIGGS. Revised Edition, Clarendon, press, oxford 1989.

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